NONLINEAR ANALYSIS OF CABLE STRUCTURES BY DYNAMIC RELAXATION METHOD

Keyword: Scilab, dynamic relaxation method, cable structure, nonlinear analysis

1. Introduction

The cable is a very important structural element. First of all, in civil engineering, where it's used in an industrial construction as bridges, geotechnical, etc., the cable present an stiffening effect with the increasing of the load, so the analysis must be nonlinear with a large displacement.

For the design of a cable structure, there are a lot of commercial software's with a *Graphic User Interface* and a simple input format. Many National Standard Codes as an example the Italian Codes (N.T.C. 2008), that require to validate the simulation through a simple structural patterns or other solvers. Using Scilab [1] is possible to validate a simulation through the internal basic function and simple manipulations matrix.

In this document I will present a software just developed to analyze cables, based on the dynamic relaxation method (DR). There are a lot of methods to analyze cable structure but DR method is the most performances than classic "FEM" analysis. In effect DR method, doesn't need to build a total stiffness matrix, because stiffness is composed from diagonal stiffness matrix of structures. The analysis is very fast because the PC memory required is less then alternative and traditional methods. In this document, DR method with *kinetic damping,* is considered an alternative method that, at each iteration, calculates the kinetic energy of an whole structural system.

In this document firstly I discussed about the Dynamic Relaxation Method, and the problem is geometrically nonlinear. I will set out the basic concept of the method with *kinetic damping*. After, the Solver will be validated through some examples taken from international bibliography.

2. The dynamics relaxation method

Analysis of cable structure require an iterative procedure because geometry of structures change at each process step. In FEM (finite elements method) global stiffness matrix must be calculated and assembled at each step with an increased time process and PC memory.

The dynamic relaxation method was originally developed to solving mathematical equations with finite difference techniques. Only after, DR was developed and used in structural engineering to analyze other structures [2]. Following the description and the formulation of the method.

DR method find the balance of the general structure damping, in the time domain, the oscillatory motion concentrated at each node. The equation governs the motion of each node in a general direction is:

$$P_{id} = [\sum K\delta]_{id} + M_{id}\delta''_{id} + C\delta'_{id} \tag{1}$$

where:

P	=	external force;
Kδ	=	internal force;
M	=	mass vector;
C	=	viscous damping coefficient;
δ'	=	velocity;
δ	=	acceleration.

Introducing vector of residual force (R_{id}), obtained from the difference between the vector of external and internal forces, previous equation become:

$$R_{id} = P_{id} - [\sum K\delta]_{id} = M_{id}\delta''_{id} + C\delta'_{id} \tag{2}$$

In the method of *kinetic damping*, the viscous damping (C) is set to zero and the procedure calculated a kinetic energy at each iteration and if its value is a maximum (peak), the procedure restarting with velocity vector set to zero.

Equation (1), becomes:

$$R_{id} = M_{id}\delta''_{id} \tag{3}$$

Expression (3) can be solved through finite difference method:

$$R^n_{id} = M_{id} \cdot \frac{\delta'^{n+\frac{1}{2}}_{id} - \delta'^{n-\frac{1}{2}}_{id}}{\Delta t} \tag{4}$$

Vector of velocity and displacement becomes so:

$$\delta_{id}^{'n+\frac{1}{2}} = \delta_{id}^{'n-\frac{1}{2}} + \frac{R_{id}^{n}}{M_{id}} \cdot \Delta t$$

$$\delta_{id}^{n+1} = \delta_{id}^{n} + \delta_{id}^{'n+\frac{1}{2}} \cdot \Delta t \tag{5}$$

At each step, the function calculate the kinetic energy of a structural system. When kinetic energy value at step *i+1* is less then value at step *i*, the "peak" of kinetic energy is between step *i* and *i+1*. The exact position of the "peak" is in function of delta (Δ) of the time [see expression (4) and (5)], so if the delta (Δ) value is small then, the position of the "peak" is more precise but, the procedure can becomes unstable. To overcome these problem, the "peak" value must be search through this expression:

$$\delta_{id}^{(t+\Delta t/2)} = \delta_{id}^{(t+\Delta t)} - \frac{3\Delta t v_{id}^{(t+\Delta t/2)}}{2} + \frac{\Delta t^2 R_{id}^{t}}{2M_{id}} \tag{6}$$

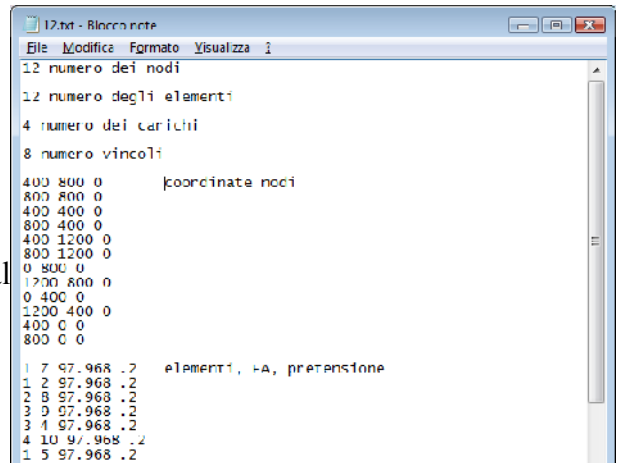
where velocity is presented with *v* symbol:

$$v_{id}^{t+\Delta t/2} = \frac{\Delta t}{2M_{id}} \cdot R_{id}^{t} \tag{7}$$

3. Software code

The software is made from three sub-programs: pre-processor, solver and post-processor. Pre-processor is a function that read data input from external files. Data input are:

- number of nodes;
- number of elements;
- number of external forces;
- number of boundary conditions;
- coordinates of each node;
- connectivity, material type and initial pre-stressed;
- external forces;
- boundary conditions.



Note 1 – Typical file input

The Post-processor's function, puts the results in a specific file, where are reported the new coordinates, the tension elements, the displacements, etc. In the graphic windows (Note 5), you can see the original's geometric plot, the deformed structure and the kinetic energy's

plot trend of structural system. Note 5 show the geometric structure and the kinetic energy trend with the localization of the peak's function of iteration.

The Solver, that implement the DR method, controls the residual error, at each iteration, through a loop function. Firstly the software determines the elastic stiffness matrix, and the geometric stiffness matrix of each element and after the construction of the mass matrix with various alternative methods. In this case the mass matrix is proportional to a ~~of~~ diagonal stiffness matrix, but there are other methods.

$$[M] = g \cdot [K] \tag{8}$$

```
//RIGIDEZZA ELEMENTI
for I=1:NM
  KE(I)=AE(I)/LO(I);
end
//INIZIO CALCOLO
while abs(ERR)>0.0001
  //RIGIDEZZA TANGENTE
  RIGID=zeros(NJ,1);
  for I=1:NM
    KG(I)=T(I)/LC(I);
    KR(I)=(KE(I)+KG(I));
  end
  for J=1:NJ
    for I=1:NM
      if (JN(I)==J | JP(I)==J)
        RIGID(J)=RIGID(J)+KR(I);
      end
    end
  end
```

Note 2 – Initial solver code

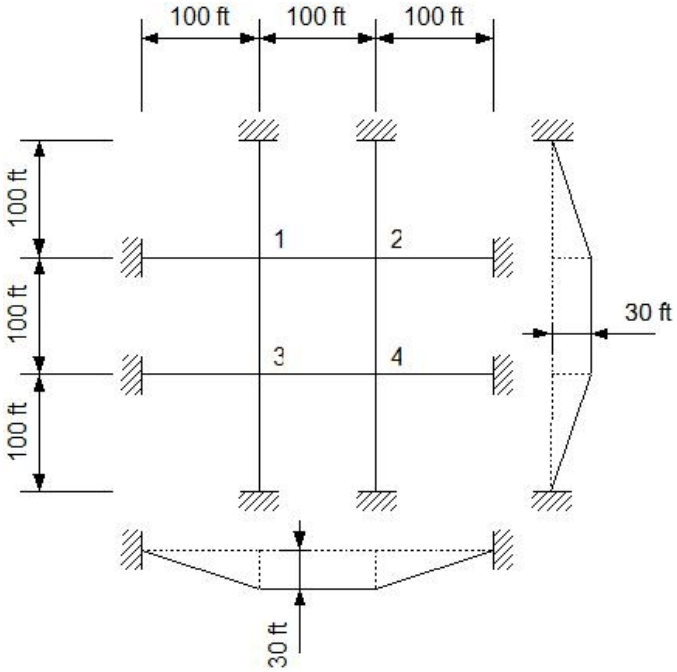
After that, the procedure calculates the residual vector, velocity vector, displacement vector and system's kinetic energy. Solver control the "peak" and after that, it updates the coordinates for a new step.

DR method, has some deficiency too, due to the values initially chosen. In effect there are a lot of coefficients that must be set before start the analysis; for example, the delta (range 1.0 – 1.1) is much more important because a high value of it can produce an unstable process. Also other coefficients as the mass, the viscous damping, etc., can condition the results.

4. Test validation

The software has been validate through a Benchmark taken from international documents and books. In this document is presented a single test taken from [3]. Note 3 show the geometric features of the structural system and the mechanical features of the cables. Test is based on a symmetric cable-net structure, with 12 nodes and 8 boundary conditions.

As shown in Note 4, the displacements are calculated by a dynamic relaxation method, comparable with the results reported in [3], where the error average is around 0.5%-1.0%.



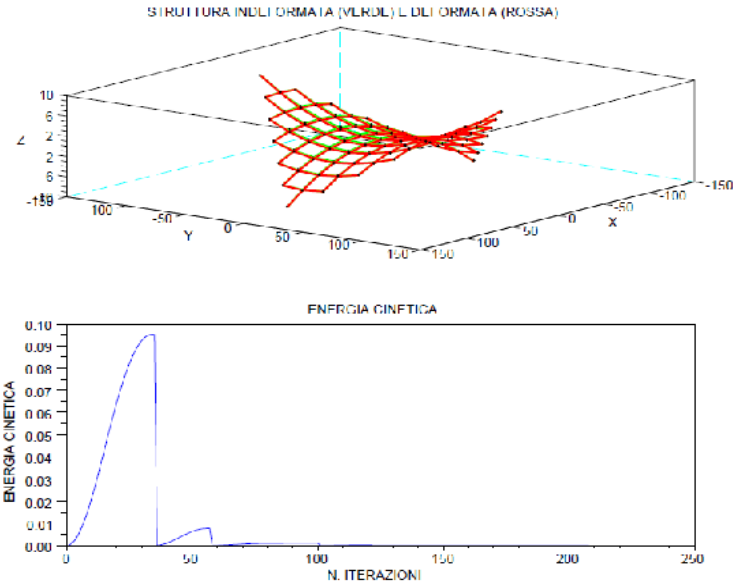
Sezione delle funi	227 in²
Modulo di elasticità equivalente	12000 kips/in²
Peso proprio delle funi	0,0001 kip/ft
Pretensione funi orizzontali	5,459 kips
Pretensione funi verticali	5,325 kips
Carico verticale ai nodi 1,2,3 e 4	8,0 kips

Note 3 – Cable-net taken from [3]

SOLUTORE	SPOSTAMENTO NODI		
	X	Y	Z
Saafan (elemento straight bar)	-0,1324	-0,1324	-1,4707
West & Kar (elemento straight bar)	-0,1325	-0,1324	-1,4698
Jayaraman & Knudson (elemento catenary)	-0,1300	-0,1319	-1,4643
Jayaraman & Knudson (elemento straight bar)	0,1322	0,1322	1,4707
G. Tibert (elemento straight bar)	0,1322	0,1322	1,4707
G. Tibert (elemento elastic parabola)	-0,1338	-0,1338	-1,4873
G. Tibert (elemento elastic catenary)	-0,1328	-0,1328	-1,4764
G. Tibert (elemento associate catenary)	-0,1338	-0,1338	-1,4874
G. Tibert (elemento elastic catenary Jayaraman & Knudson)	-0,1328	-0,1325	-1,4760
Metodo delle rigidezze	-0,1343	-0,1343	-1,4730
Metodo del rilassamento dinamico	-0,1343	-0,1343	-1,4730
Energia potenziale	-0,1335	-0,1336	-1,4647
Metodo delle rigidezze modificato	-0,1343	-0,1343	-1,4730

Note 4 – Comparison displacements

The Comparison displacement was made with other finite element formulation to a nonlinear analysis for the large displacements. More test validation are shown in [4].



Note 5 – Graphic output

5. Conclusion

This document presented a program developed in Scilab Platform [1] for the analysis of cable structure by dynamic relaxation method with kinetic energy damping.

After a general describing of DR method with kinetic damping and source code, the procedure has been validate with a structure taken from international documents [3].

The software can help the engineers to validate complex analysis as Italian Standard Code N.T.C. 2008 or even used to analyze general cable structures with external *Graphical User Interface*.

6. Reference

[1] Scilab 5.2, Scilab Consortium, 2009

[2] W.J. Lewis. Tension structures, Thomas Telford, 2003

[3] Gunnar Tibert: Numerical analysis of cable roof structures, Licentiate Thesis, 1999

[4] Casagrande E.: Introduzione all'analisi automatica di sistemi strutturali composti da funi. Cleup, 2011

7. Reference

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